INVESTIGATING NONLINEAR VISCOELASTIC PROPERTIES OF BRAIN TISSUE USING THE FORCED VIBRATION METHOD

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INTRODUCTION

The objective of this study was to develop a method for characterizing nonlinear viscoelastic properties of brain tissue using the forced vibration method. Understanding nonlinearity of brain material is crucial in studying the effects of a) finite deformation, b) multiple impacts and c) shock and acceleration waves in traumatic brain injury.

Brain tissue, like most other soft tissues, exhibits viscoelastic behavior. Traumatic brain injury is generally caused by deformation impulses with duration of a few milliseconds. Therefore, it is necessary to use dynamic test methods for characterizing the brain material. The study of dynamic viscoelasticity of brain tissue began in late 1960's. The reader should see Arbogast and Margulies (1998) for a brief review of the previous dynamic test results. In previous studies the brain material is assumed to be linearly viscoelastic. However, the more recent results of stress relaxation tests on human and bovine brain (Takhounts, 1998) and on porcine brain (Prange et al., 1998) show that the material instantaneous and time dependent responses are generally nonlinear. In addition injury tolerance of neural fibers is about 15% to 20% tensile strain or 20% to 30% shear strain (Thibault et al., 1990). The study presented in this paper accounts for both material and geometric nonlinearities in characterizing brain tissue.

BRIEF THEORY

In the Green-Rivlin model with multiple hereditary integrals, the stress response is considered to be nonlinear with respect to both strain and time. Developing a fully nonlinear model provides a basis for evaluating simpler models such as linear and quasilinear viscoelastic models. In this study the brain material was assumed to be isotropic, homogeneous and incompressible and a third order Green-Rivlin constitutive relation following Pipkin (1964) was used. Assuming simple shear deformation with \( x_1 = X_1 + K(t)X_2, x_2 = X_2, x_3 = X_3 \), the sample shear stress can be written as:

\[
\Sigma_{12} = \frac{1}{2} \int_0^t \psi_2 K(\tau_1) d\tau_1 + \frac{1}{2} \int_0^t \psi_6 K(\tau_1)K(\tau_2) d\tau_2 + \frac{1}{8} \int_0^t \psi_{III} K(\tau_1)K(\tau_2)K(\tau_3) d\tau_3
\]

where \( \Sigma_{12} = \sigma_{12} + \frac{1}{2} K \Delta \sigma_n - \frac{1}{2} K^2 \sigma_{12} - \frac{1}{8} K^3 \Delta \sigma_n \), with \( \Delta \sigma_n = \sigma_{11} - \sigma_{22}, \sigma_{ij} \) is the Cauchy stress tensor, and \( \psi_2, \psi_6 \), and \( \psi_{III} \) are the time dependent material kernel functions to be determined.

MATERIAL AND METHOD

Small cylindrical samples (about 10 mm to 15 mm diameter and 2 mm to 5 mm length)
of white and gray matters of fresh bovine brain tissues were tested using the experimental apparatus developed for this purpose and described in Darvish et al. (1998). Samples were cut along the superior-inferior direction of the head. During all tests, samples were immersed in artificial cerebrospinal fluid that was kept at the body temperature. In order to investigate nonlinearity of the constitutive relation (1), the input excitation was selected as a superposition of three harmonic inputs:

\[ K(t) = K_1 \sin(\omega_1 t) + K_2 \sin(\omega_2 t) + K_3 \sin(\omega_3 t) \]  

(2)

with a total shear strain level up to 30%. In the frequency range below the first natural frequency (about 400-450 Hz) where the inertial effect is small, the output force signals were used to measure \( \sigma_{12} \) and \( \sigma_{22} \) (\( \sigma_{11} = 0 \)) and subsequently to derive the shear complex moduli \( G_n(\omega_1,..,\omega_n) \) for \( n = 1, 2, 3 \) (Lockett, 1972). Due to nonlinearity, the force responses had frequency components in addition to the fundamental frequencies \( \omega_1, \omega_2, \) and \( \omega_3 \). By performing spectral analysis on the force signals and using the magnitude and phase of the first three harmonics and two-term and three-term combinations of the fundamental frequencies, the first-, second- and third-order complex shear moduli were determined. The presence of non-integer harmonics in the force signals could not be modeled with single hereditary integral models and confirmed the Green-Rivlin model assumption. In order to determine the kernel functions in time domain that can be used in explicit numerical algorithms a product form was assumed for the second- and third-order relaxation kernels (Nakada, 1960). The first-order relaxation functions were fitted to discrete spectrum approximation in the form of 4-term Prony series with short-, intermediate-, and long-time decay rates in the order of magnitude of 10 ms, 100 ms, and 1.0 s respectively in addition to a constant term.

**SUMMARY**

In this study, a method was presented to develop nonlinear viscoelastic model of brain tissue in the form of multiple hereditary integrals using the forced vibration method. The results show that a third-order Green-Rivlin material model sufficiently describes the nonlinearity of the brain material subject to finite shear deformation.

**REFERENCES**


