ESTIMATION OF THE FINITE CENTER OF ROTATION IN PLANAR MOVEMENTS

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INTRODUCTION

Human movement is not generally two-dimensional in nature, despite this human movement is often analyzed in two-dimensions due to the relative simplicity of data collection, analysis, and interpretation in two-dimensions. The center of rotation for human joints in two-dimensions is a kinematic variable which is popular for the assessment of joint function. The center of rotation is normally represented by the finite center of rotation (FCR) which relates to a measure taken from a single finite displacement.

A number of different procedures have been advanced for the computation of the FCR. It was the purpose of this study to compare the accuracy of the procedure of Crisco et al. (1994) for estimating the FCR with a new procedure which uses least-squares principles.

METHOD

Two different procedures were used to compute the FCR, the procedures were,
Procedure A - the procedure of Crisco et al. (1994) who presented the formulae for determining the FCR for a rigid body undergoing a finite displacement.
Procedure B - this uses least-squares principles to determine rigid body attitude and position. The first step is to compute the mean position vectors,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

where x - position of a point measured on the body in a body fixed reference frame, y - position of the point measured in the inertial reference frame, and n is the number of points measured. The positions of the points are computed relative to the mean position vector

$$\hat{x}_i = x_i - \bar{x} \qquad \hat{y}_i = y_i - \bar{y}$$

The attitude angle (ϕ) of the body can then be estimated in a least-squares sense from

$$\phi = -tan^{-1}\left(P/Q \right)$$

Where $P = \sum_{i=1}^{n}\left(\hat{y}_{xi}\hat{x}_{yi} - \hat{y}_{yi}\hat{x}_{xi} \right)$,

$Q = \sum_{i=1}^{n}\left(\hat{y}_{xi}\hat{x}_{xi} + \hat{y}_{yi}\hat{x}_{yi} \right)$ and the subscripts x, y refer to the horizontal and vertical components of the vectors respectively. The position vector of the body v can be determined from the mean vectors

$$v = \bar{y} - [\, R(\phi)\,]\bar{x}$$

If the attitude and position are known at two instants (e.g. t_1, t_2), then it is possible to compute the FCR from

$$FCR = p + \left[2.tan\left(\tfrac{1}{2}\varphi\right) \right]^{-1} . \left[R\left(90^o\right) \right] . \Delta v$$

Where $p = \frac{1}{2}\left(v(t_1) + v(t_2) \right)$, $\varphi = \phi(t_2) - \phi(t_1)$, $\left[R\left(90^o\right) \right]$ - matrix representing a 90° counter-clockwise rotation, and $\Delta v = v(t_2) - v(t_1)$.

Three different evaluations were made of the two procedures. In all cases the FCR had to be estimated, and was assumed to be the origin of the inertial reference system.

1) Two data points 60 mm apart aligned so that a line between them lies on the y axis, with the mid-point at the origin of the axis system. Noisy versions of these data points were rotated through angles from 4 to 24 degrees.
2) A set of similarly orientated data points but with the distance from the FCR to the mid-point of the pair varied from 0 to 180 mm, the rotation angle was fixed at 10 degrees.
3) Using the same data points as in evaluation 1, with a fixed rotation angle of 10 degrees, but with the standard deviation of the noise varying from 0.1 mm to 1.5 mm.

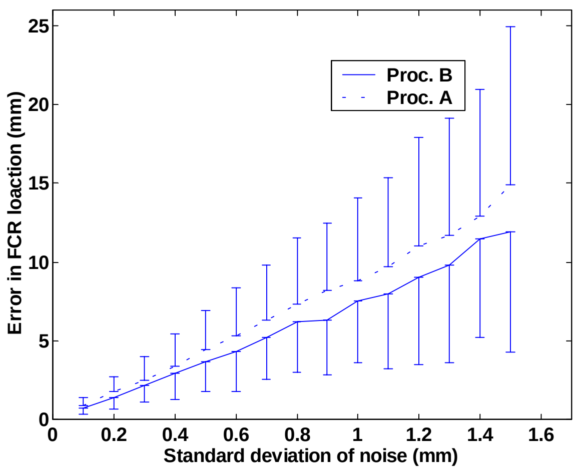

Figure 1 – Results for evaluation 1.

As the mean position of a pair of landmarks was moved further from the FCR the accuracy of its determination by both procedures decreased, with the better performance by procedure A. With increasing noise levels the accuracy with which the FCR was estimated decreased. Procedure A produced higher mean errors than procedure B (figure 2).

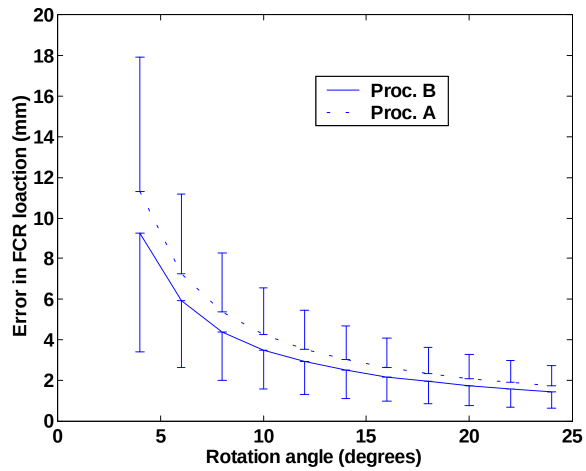

Figure 2 – Results for evaluation 3.

DISCUSSION

For the comparisons made of the two procedures, procedure B under all conditions was superior to procedure A, albeit by small amounts under certain conditions. Crisco et al. (1994) demonstrated that their procedure was more accurate than a number of available procedures (e.g. Reuleaux, 1875; Spiegelman. and Woo, 1987). Procedure B used least-squares principles to estimate the FCR, which removed some of the influence of noise in the position data, but could not account for all of it. Careful experimental protocols to reduce noise are important irrespective of which procedure is used.

REFERENCES

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