The Spanning Set Defines Variability in Locomotive Patterns
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INTRODUCTION

Evaluation of the standard deviation of a mean ensemble curve has been used to describe the amount of variability in locomotive patterns (Dingwell et al., 1999; Hamill et al., 1999; Winter, 1983). However, few investigations that have utilized such an approach have found statistical differences in variability present during locomotion. In the present study, we suggested that the standard deviations about the mean ensemble curve represented the spanning set for all possible neuromuscular solutions. The greater distance between the two vectors that define the spanning set, the more variability within the locomotive pattern. Thus, our purpose was to present an alternative method for defining variability in locomotive patterns, and to compare this method to traditional measures of variability.

METHODS

A healthy subject ran on a treadmill at a self-selected pace while barefoot and with footwear. Ten consecutive footfalls were analyzed for each condition. Right side, sagittal kinematic data was collected with a 180 Hz camera. Positional coordinates were smoothed using a Butterworth Low-pass Filter with a selective cut-off algorithm (Jackson, 1979). The cut-off frequency values used were 13-16 Hz. Angles of the thigh, and shank were calculated with respect to the right horizontal. An absolute approach was used to calculate the knee joint angle ($\theta_{\text{Knee}} = \theta_{\text{Thigh}} - \theta_{\text{Shank}}$). Mean ensemble curves for the stance period were created for the knee joint for each condition. Measures of variability during the stance period were calculated using the Coefficients of Variation (CV) (Winter, 1983) and the Mean Deviation (MD) (Hamill et al., 1999).

\[
CV = \frac{1}{N} \sum_{i=1}^{N} \frac{s^2_i}{X_i} \quad \text{MD} = \left( \frac{1}{N} \sum_{i=1}^{N} |S_i| \right) / N
\]

Where $N =$ number of samples (100), $X_i =$ average angle at the $i$th point, and $S_i =$ standard deviation about $X_i$. The spanning set was defined by initially utilizing a least squared method to fit seventh order polynomials to the respective standard deviation curves. Coordinate mapping was used ($f \rightarrow [f]_B$) to introduce a familiar coordinate system that could be utilized to describe the properties of the polynomials in $\mathbb{R}^n$. Since $\mathbb{P}_n$ is isomorphic to $\mathbb{R}^{n+1}$, the coefficients from the respective polynomials were used to map a vector space that could be used to define vectors in the spanning set. Coordinate mapping was possible because the spanning set formed a basis (linearly independent set). Since $[f]_B$ was a basis, the coordinate mapping resulted in a 1:1 relationship, and accurately described the properties of the polynomials.
Table 1. Measures of variability used to describe knee joint during stance.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Footwear</th>
<th>Barefoot</th>
</tr>
</thead>
<tbody>
<tr>
<td>CV (%)</td>
<td>4.37</td>
<td>6.42</td>
</tr>
<tr>
<td>MD (deg)</td>
<td>2.35</td>
<td>2.85</td>
</tr>
<tr>
<td>Magnitude of the Spanning Set</td>
<td>7.26</td>
<td>20.0</td>
</tr>
</tbody>
</table>

The magnitude of the spanning set was determined by calculating the norm of the difference between the two vectors of the spanning set.

RESULTS AND DISCUSSION

Graphically, differences in the variability between the two conditions were evident during the stance period (Figure 1). While CV and MD revealed larger values for the barefoot condition, these differences were small (Table 1). It is possible, that previous investigations that have used such traditional measures, were not able to show significance due to such small differences (Dingwell et al, 1999; Hamill et al., 1999). The magnitude of the spanning set showed larger differences in variability between the two conditions (Table 1). It seems that compared with the other measurements, the magnitude of the spanning set was a more sensitive measure of variability. Furthermore, the magnitude of the spanning set coincides better with the graphic observation that the barefoot condition was more variable (Figure 1).

CONCLUSIONS

The usage of the spanning set is a novel method of quantifying variability in locomotive patterns. Based on our data, this approach may be able to provide a sensitive measure of differences in variability during locomotive patterns. Future investigations may want to utilize this method to quantify variability in locomotive patterns.

REFERENCES


![Figure 1. Mean ensemble curves (dashed line) and standard deviations (bold line) for the knee joint from the respective conditions: A.) Barefoot, B.) Footwear.](image-url)