CONTROLLING BIFURCATIONS AND CHAOTIC GAIT WITH HIP JOINT ACTUATIONS IN A SIMPLE WALKING MODEL

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INTRODUCTION

Current investigations have indicated that human walking has a chaotic structure (Buzzi et al., 2003). In addition, Garcia et al. (1998) developed a passive dynamic walking model that exhibits period doublings that appear to lead to chaos (Garcia et al., 1998). Recently, we have replicated Garcia et al.’s work and have confirmed the presence of chaos through a rigorous exploration of the Lyapunov Exponents (LyE) of the model’s gait (Kurz and Stergiou, 2003). In this present investigation we added a torsional hip spring actuator to the model that provided a burst of energy to the swing leg during gait. Our inspiration for adding a hip joint actuator was based on previous investigations that have indicated that parametric forcing could be used to control bifurcations in a pendular apparatus (Starrett & Tagg, 1995). We speculated that hip joint actuation may provide similar forcing that would allow for the control of chaotic gait in the model.

Thus, the purpose of this investigation was to explore the effect of hip joint actuation on the control of the dynamics of the model.

METHODS

The passive dynamic gait model consists of two rigid legs connected by a frictionless hinge at the hip (Figure 1). We supplied actuation of the hip joint using a torsional spring. Energy was supplied to the model via a sloped walking surface. Two coupled second-order differential equations were used to define the behavior of the legs. The equations of motion were simplified by assuming that the mass of the model was concentrated at the hip.

\[ \ddot{\theta}(t) - \sin(\theta(t) - \gamma) = 0 \]

\[ \ddot{\theta}(t) - \ddot{\phi}(t) + \ddot{\theta}(t)^2 \sin \phi(t) - \cos(\theta(t) - \gamma) \sin \phi = k \phi(t) \]

where \( \theta \) is the stance leg angle, \( \phi \) is the swing leg angle, \( k \) is the normalized torsional spring constant, and \( \gamma \) is the ramp angle. Simulating the model consisted of integrating the equations of motion and applying a transition rule at heel-contact. Similar to Garcia et al. (1998), we modeled heel-contact as instantaneous and perfectly inelastic.

The model was simulated for 5000 steps at each ramp angle with no hip joint actuation (\( k=0 \text{ s}^{-2} \)). The first 500 steps of the simulation were discarded. Poincaré sections were used to confirm the order of the gait simulations (Figure 2). The Poincaré section was composed of the angular displacement vs. the angular velocity for the initial conditions of the stance leg for each step in the simulated gait. An increased number of orderly points in the section indicated higher order gaits. LyE were calculated from the data used to create the Poincaré sections to confirm the presence of chaos. A positive LyE value indicated a chaotic gait pattern.
Once all period-n gaits were identified, we systematically increased hip joint actuation at each of the period-n gaits. The effect of hip joint actuation was determined through inspection of the Poincaré sections.

RESULTS AND DISCUSSION

With no added hip joint actuation, the model had a cascade of bifurcations that lead to a chaotic gait pattern as $\gamma$ was increased (i.e. period-one, period-two, period-four, etc.). A chaotic gait pattern was present for slopes of $0.01839 \text{ rad} < \gamma < 0.019 \text{ rad}$ (LyE range = +0.002 to +0.158). Our simulations also indicated that hip joint actuation was a mechanism to control the cascade of bifurcations that lead to chaos in our walking model. As the hip joint actuation was increased, the order of the period-n gait at $\gamma$ was decreased. For example, a period-8 gait was reduced in order to a period-4 gait to period-2 gait to period-1 gait as the hip joint actuation was systematically increased (Figure 2). Our simulations suggest that altered hip joint actuation may be related to modifications in the chaotic structure of gait patterns previously seen in pathological populations (Buzzi et al., 2003).

Adjustments in hip joint actuation appeared to be most beneficial in regions where the model was not previously able to walk. Without the addition of hip joint actuation ($k=0 \text{ s}^{-2}$) the model would fall down at ramp angles larger than 0.019 rad. However, the addition of hip joint actuation allowed the model to walk with a chaotic gait at ramp angles that were previously considered unstable. Buzzi et al. (2003) suggested that a chaotic gait is more stable due to greater flexibility in the gait pattern. We suggest that modifications in hip joint actuation may be a mechanism to help the gait pattern remain in the chaotic basin of attraction where the neuromuscular system can take advantage of such flexibility.

SUMMARY

Simulations of the walking model indicate that hip joint actuation is a mechanism to control the structure and stability of chaotic gait. These simulations provided a novel mechanical approach to explain how the biological system functions to control chaotic gait.

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REFERENCES


Figure 2: Poincaré sections for the model while walking at $\gamma = 0.01823 \text{ rad}$ and $k=0 \text{ s}^{-2}$ (A), $k=0.001 \text{ s}^{-2}$ (B), $k=0.01 \text{ s}^{-2}$ (C), $k=0.06 \text{ s}^{-2}$ (D).