INTERPOLATING THREE DIMENSIONAL KINEMATIC DATA USING QUATERNIONS

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INTRODUCTION

Multiple CT volume images can be used to noninvasively determine the normal and pathological kinematics of the carpal bones of the wrist (Crisco et al. 1999). Only a limited number of positions can be acquired with CT due to radiation exposure. Accurate interpolation of the kinematics between these positions permits a more thorough kinematic analysis, especially of *in vivo* data sets where it is difficult to precisely control an independent position variable. Quaternions, four-dimensional unit vectors describing rotations, are currently used in computer graphics to interpolate a smooth path between key frames. We report a method for interpolating any 3-D kinematic data set by combining cubic quaternion splines with Catmull-Rom interpolation for translations.

MATERIALS AND METHODS

A quaternion spline (Shoemake 1985, Kim 1995), coupled with Cartesian Hermite curve was implemented and examined using a test object and *in vivo* kinematic data.

Image and Data Acquisition: Gold standard data for verification was acquired using five sensors tracking a rigid body motion, collected with an Optotrak (Northern Digital, Waterloo, ON) sampling at 30 Hz. Test data was resampled at 6 Hz from the gold standard and interpolated. Ten distinct wrist positions of a subject were scanned using a GE Highspeed Advantage CT (GE Medical Systems, Milwaukee, WI). Kinematic data of each carpal bone were calculated using established markerless bone registration techniques (Crisco 1999).

C^1-continuous Piecewise Quaternion Spline: Quaternions are elements of the four-dimensional space *Q* formed by the real axis and three imaginary orthogonal axes, **i**, **j**, and **k** that obey Hamilton's rule:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1 \quad \text{Eq. 1}$$

Each four-component quaternion is written as a unit vector:

$$\mathbf{q} = [\cos\theta \,, \mathbf{n} \sin\theta] \quad \text{Eq. 2}$$

where **n** is the orientation of the helical axis of motion (HAM) and θ is twice the angle of rotation, ϕ, about that axis. Quaternions are therefore directly calculable from 3x3 Euler rotation matrices.

An interpolative spline was implemented using a cubic curve between quaternions with an iterative spherical linear interpolation (slerp) technique (Shoemake 1985). For compactness *slerp*($\mathbf{q}_0$, $\mathbf{q}_1$; t) is notated $(\mathbf{q}_0 : \mathbf{q}_1)_t$ and is defined as:

$$\mathbf{q}(t) = (\mathbf{q}_0 {:} \mathbf{q}_1)_t = \mathbf{q}_0 (\mathbf{q}_0^{-1} \mathbf{q}_1)^t \quad \text{Eq. 3}$$

for $0 \le t \ge 1$ where *t* is the fractional advance from $\mathbf{q}_n$ to $\mathbf{q}_{n+1}$.

According to this method the tangents, $\mathbf{a}_n$ and $\mathbf{b}_{n+1}$, at each quaternion, $\mathbf{q}_n$, must be calculated. Cyclic motions have tangents for all points, but single direction motions encounter discontinuities at endpoints. A simple end-point tangent solution (Kim 1995) was implemented. (Fig. 1)

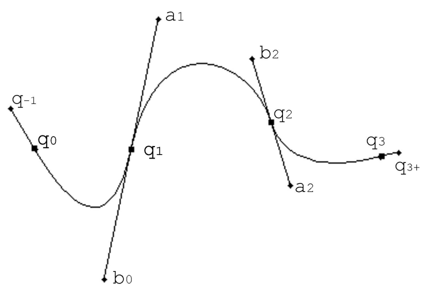

Fig. 1: An example curve showing the location of tangents $\mathbf{a}_n$, $\mathbf{b}_n$, $\mathbf{q}_{-1}$, and $\mathbf{q}_{3+}$.

The final equation for the cubic spline is:

$$(\mathbf{q}_0 : \mathbf{a}_0)_t = \mathbf{p}_1$$
$$(\mathbf{p}_1 : \mathbf{p}_2)_t = \mathbf{p}_4$$
$$(\mathbf{a}_0 : \mathbf{b}_1)_t = \mathbf{p}_2 \qquad (\mathbf{p}_4 : \mathbf{p}_5)_t = \mathbf{q}_0(t)$$
$$(\mathbf{p}_2 : \mathbf{p}_3)_t = \mathbf{p}_5$$
$$(\mathbf{b}_1 : \mathbf{q}_2)_t = \mathbf{p}_3 \qquad \text{Eq. 5}$$

C^1- continuous Piecewise Cartesian Spline: Hermite curves, specifically Catmull-Rom curves, are well-described mathematical tools for interpolating smoothly between any set of three-dimensional points. After quaternion interpolation, the remaining HAM parameters (axis location in space, $\mathbf{Q}_L$ and translation along the axis, $\mathbf{t}_{ham}$) were splined using these curves. $\mathbf{t}_{ham}$ was interpolated directly. $\mathbf{Q}_L$ was calculated to minimize its displacement in the data set. The location, on the axis, nearest the origin was found for the first HAM, $\mathbf{Q}_{L1}$. Each subsequent axis location, $\mathbf{Q}_{Ln}$, was calculated to be the point closest to $\mathbf{Q}_{L1}$.

Combining Quaternion and Hermite Curves: Hermite curves operate in a linear space whereas quaternion curves operate in a spherical space. This causes a discrepancy in the length of each interpolated element. To compensate we interpolated the quaternions using a constant increment t, then constructed the Hermitian parameter t_n:

$$t_n = \frac{\sum \theta_n}{S} \qquad \text{Eq. 6}$$

where S is the quaternion segment arc length and θ_n is arc length for increment n.

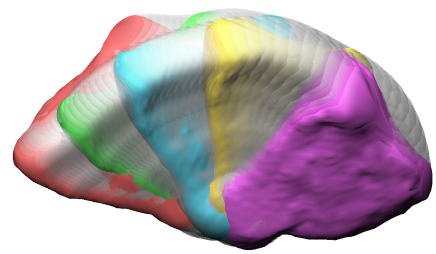

Fig. 2: Scanned capitate positions (colors) and interpolated positions (grey).

RESULTS AND DISCUSSION

- C^1-continuity was verified for all interpolated curves made from test data.
- When applied to the *in vivo* data set, bones maintained anatomically reasonable positions.
- Curves obtained from these splines allowed for selection of a constant independent variable across subjects.
- Large changes in data point increments cause artifacts from varied arc lengths.
- Interpolations are directionally dependent due to differences in tangent calculation.

Comparison of “gold standard” and “test data” HAM parameters yielded mean differences of 0.01° ± .06° in ϕ, 0.3 ± 1.8mm in $\mathbf{t}_{ham}$, 0.55° ± .8° and 4.5 ± 7.4mm in HAM orientation and location.

With the mentioned limitations, this method gives an accurate path between a data set of kinematic transforms sampled at irregular intervals.

REFERENCES

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ACKNOWLEDGMENTS

Funded in part by NIH AR44005