

AN INDEX FOR EXAMINING MOMENT INDUCED ACCELERATIONS

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INTRODUCTION

The moments generated by a muscle are capable of accelerating joints in addition to any it crosses (Zajac & Gordon, 1989). One way of examining this effect is to perform an induced acceleration analysis. Kepple et al. (1997) experimentally computed the resultant joint moments during human gait, and then computed the angular accelerations these moments induced. Zajac et al. (2003) also examined gait, but used a simulation model which estimated the forces produced by the muscles to generate gait, the angular accelerations induced by these muscle forces were computed.

The purpose of this study is to introduce a induced acceleration index (IAI) which permits quantification of the relative potential of accelerations produced at joints in the system to accelerate other joints in the kinematic chain.

THEORY

For a system of articulated rigid bodies the equations of motion can be written in matrix form, Craig (1989), as follows,

$$T = M(\theta)\ddot{\theta} + v(\theta, \dot{\theta}) + G(\theta) \quad (1)$$

where T - vector of joint moments, $M(\theta)$ - inertia matrix, $v(\theta, \dot{\theta})$ - vector of centrifugal and Coriolis terms, $G(\theta)$ - vector of gravity terms, $\theta, \dot{\theta}, \ddot{\theta}$ - generalized coordinates and their first and second derivatives.

For an induced acceleration analysis accelerations caused by the centrifugal, Coriolis, and gravity terms are ignored as the focus is the accelerations caused by the moments acting at the joints. In T the moments are set to zero except for those at the joints considered active, and the acceleration computed at the other joints, the induced accelerations. For simplicity the joints will be classified as active for those considered to be generating a moment, and inactive for those not generating a moment but with the potential to be accelerated by the active joints. For this analysis equation 1 is rewritten as,

$$\begin{bmatrix} T \\ [0] \end{bmatrix} = [M] \begin{bmatrix} \ddot{\theta}_A \\ \ddot{\theta}_I \end{bmatrix} = \begin{bmatrix} M_{AA} & M_{IA} \\ M_{AI} & M_{II} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_A \\ \ddot{\theta}_I \end{bmatrix} \quad (2)$$

where $\ddot{\theta}_A$ - vector of accelerations at active joints, $\ddot{\theta}_I$ - vector of accelerations induced at inactive joints, M_{AA} - sub-matrix relating active moments to active joints accelerations, the other sub-matrices are similarly named, T_A - vector of moments applied at active joints, $[0]$ - vector of zeros.

The induced accelerations at the inactive joints can be computed from,

$$M_{AI}\ddot{\theta}_A + M_{II}\ddot{\theta}_I = [0] \quad (3)$$

Therefore,

$$\ddot{\theta}_I = -M_{II}^{-1}M_{AI}\ddot{\theta}_A \quad (4)$$

so a matrix (M_{IAI}) can be defined which dictates the induced accelerations,

$$M_{IAI} = M_{II}^{-1}M_{AI} \quad (5)$$

This matrix is a function of the joint angles and segmental inertial properties only. The

inertia matrix is symmetric and positive semi-definite, and the sub-matrices maintain these properties so are always invertible. If one joint only is active this matrix, M_{IAI} , is a scalar and a number can be produced which shows for a given system how the induced accelerations vary with joint angles. If more than one joint is considered active the condition number of the matrix can be used as the index (Lawson & Hanson, 1974).

METHOD

To illustrate the IAI the upper limb was examined for planar motion. For 11 subjects anthropometric data were collected. These data were used to model the upper arm, forearm, and hand as series of geometric solids. From this modeling the inertial properties of the segments were determined, and used to compute the IAI for the shoulder and elbow joints. The equations for the IAI for these joints depends on the elbow joint angle only, which was varied from full extension to full flexion.

RESULTS

The IAI for elbow demonstrates the shoulder can cause significant angular acceleration of the elbow joint (figure 1). In contrast elbow moments have relatively small influences on the shoulder angular acceleration. The ability to accelerate an adjacent joint is greatest when these joints are near to fully extended.

DISCUSSION

In human movement the ability of muscles to cause significant acceleration at joints they do not cross is exploited to a large extent (Zajac et al., 2003). The proposed

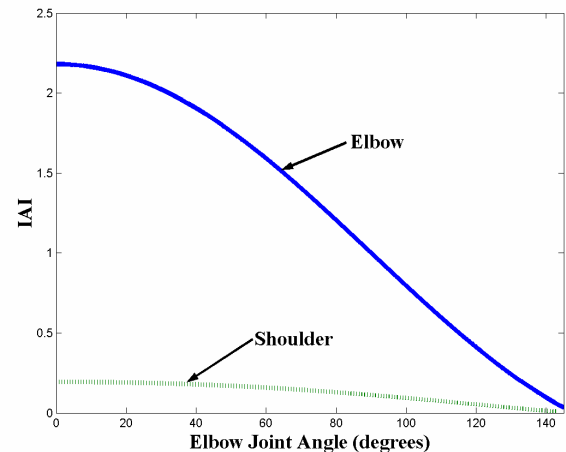


Figure 1: IAI for the shoulder and elbow joints.

induced acceleration index permits quantification of the relative potential of accelerations produced at joints in the system to accelerate other joints in the kinematic chain. The index can also be computed to examine how this coupling may change due to inertial changes to the system, for example if the inertial properties of a prosthetic are changed, or if a tool is held in the hand.

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