

WHEN SOME GAIT DETERMINANTS FAIL, SOME EQUATIONS CAN EVENTUATE

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INTRODUCTION: Saunders, et al. (1953) posed a “compass gait” model of human walking in which the leg of the compass represented the human leg length “ L ”. We have used a similar model (Fig. 1), except we included a rocker base to represent the roll-over shape of the foot/ankle rocker system (Hansen, et al. 2000). To an approximation, this modification leads to a longer equivalent leg length “ L_v ”, which we call the virtual leg length ($L_v \cong 1.7 \cdot L$).

This new model has been useful in deriving equations that relate a number of walking parameters (Gard & Childress, 2001).

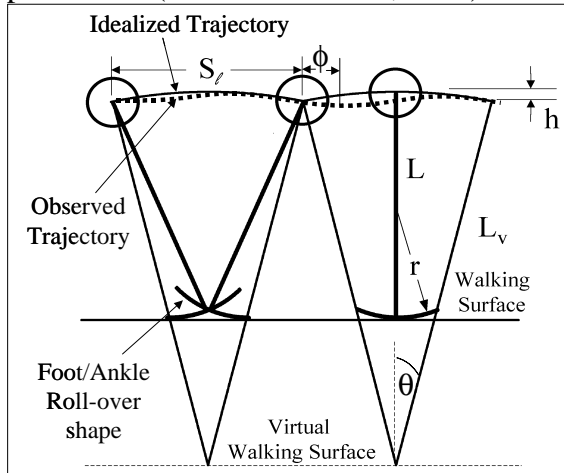


Figure 1: Rocker-based inverted pendulum model.

In opposition to the six determinants of gait, Gard and Childress (1997 & 1999) have found that pelvic obliquity and stance-phase knee flexion do not decrease vertical trunk excursion. Since these 2nd and 3rd determinants fail to alter the peak to peak vertical excursion of the body, and since our idealized model has been found to reasonably predict relationships between

walking parameters, we have used “ h ” (see Fig. 1), as derived from the idealized model, to be the peak-to-peak value of a sinusoidal equation representing the trajectory actually observed, which appears sinusoidal with a phase shift. Other equations eventuate, one that relates to the kinetics of walking. This equation, Eq. 2, appears useful because it provides quantitative relationships between vertical acceleration and various gait parameters.

METHODS: The method is mainly to mathematically manipulate equations that come from relations and approximations concerning non-disabled human walking. We know from Koopman (1989) and others

that $S_l = a \cdot f_{cad}$ and $f_{cad} = \sqrt{\frac{V}{a}}$ where f_{cad} is the cadence, a is the step ratio constant, S_l is the step length and V the walking speed. Using simple trigonometric relationships, small angle approximations for θ , and an approximation for the cycloidal trajectory of the rocker-based inverted pendulum (Morawski & Wojcieszak, 1978) it can be shown that

$$h = \frac{a \cdot V}{8 \cdot L_v}$$

Substituting $h/2$ into a sinusoid function we obtain an estimate of vertical position “ y ”.

$$y \cong -\frac{a \cdot V}{16 \cdot L_v} \cdot \cos(\omega \cdot t - \mathbf{f}),$$

where \mathbf{f} is the angular shift between the observed trajectory and the first harmonic of the idealized trajectory. Differentiating the vertical position twice and substituting

$$\mathbf{w} = 2 \cdot \mathbf{p} \cdot f_{cad} = 2 \cdot \mathbf{p} \cdot \sqrt{\frac{V}{a}}, \text{ we obtain}$$

Eq. 1 for the vertical acceleration.

$$\ddot{y} \cong \frac{\mathbf{p}^2 \cdot V^2}{4 \cdot L_v} \cdot \cos(\mathbf{w} \cdot t - \mathbf{f}) \quad \text{Eq. 1}$$

If “g” is the gravitational constant, the acceleration “Acc” may be written as:

$$Acc \cong g + \frac{\mathbf{p}^2 \cdot V^2}{4 \cdot L_v} \cdot \cos(\mathbf{w} \cdot t - \mathbf{f}) \quad \text{Eq.2}$$

If walking makes a transition to jogging when $Acc = 0$, then V_T , the transition speed, can be estimated by Eq. 3.

$$V_T = \frac{2}{\mathbf{p}} \cdot \sqrt{g \cdot L_v} \quad \text{Eq. 3}$$

RESULTS AND DISCUSSION: A

summed GRF for one subject from our lab’s data files is shown in Fig. 2. The amplitude of the sinusoid is approximated as (mg – minimum). The GRF above “mg” is not very sinusoidal in shape but the impulse magnitude above “mg” equals that below. Data at three walking speeds is plotted in Fig. 3. A curve predicted by theory is shown along with the data. The preliminary data tend to be in agreement with the theory.

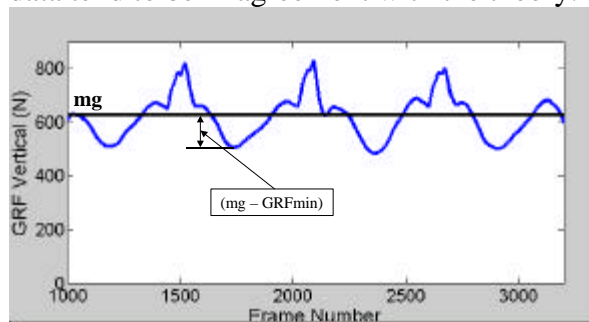


Figure 2: Vertical Ground Reaction Force.

SUMMARY: An interesting finding, if supported by more data, is that the peak vertical GRF increases about the body weight (mg) as the square of the walking speed. Sinusoidal analysis may be important to walking because it avoids the need for initial conditions. Some numeric results are that an 64 kg person with height of 1.73 m,

leg length of 0.53 * height, $L_v = 1.7 L$, walking at 1.0 m/s 1.4 m/s, 1.8 m/sec and 2.5 m/s will have peak forces above “mg” of about 16%, 31%, 52%, and 100% respectively. Actual peak forces may be somewhat greater. The transition speed V_T is predicted by the model to be 2.49 m/s.

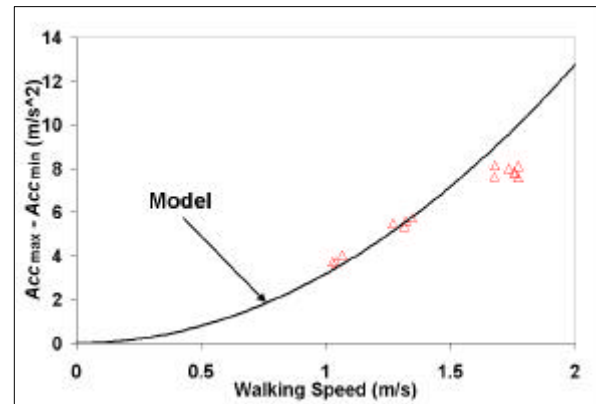


Figure 3.

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