ANALYSIS OF FATIGUE DAMAGE IN BOVINE TRABECULAR BONE

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INTRODUCTION
Cyclic loading of trabecular bone can lead to fatigue damage and increased risk of fracture in both young and the elderly populations. Fatigue damage processes in trabecular bone include reduction of elastic modulus and accumulation of residual strain. Development of models relating the above effects to the loading variables (stress, strain) is essential for a predictive understanding of the process. This study analyzes the effect of stress and strain variation on fatigue damage processes in bovine trabecular bone, and develops a model relating fatigue life to the stress level.

METHODS
Bovine trabecular bone specimens were tested under monotonic and cyclic compression loadings (Moore, 2002, 2003). For monotonic tests, the initial modulus (E₀) for each specimen was determined by fitting a linear curve to the initial portion (ε = 0.001 to 0.004) of the compressive stress-strain curve. The secant moduli (E_{sec}) of the specimens at higher strains were calculated and normalized by the initial modulus. For cyclic compression tests, the initial modulus (E₀) was measured by taking the slope of the best linear fit of the final preconditioning loading cycle (ε = 0.001 to 0.003). The cyclic stress-strain curves were divided into 4 ranges of maximum normalized stress (Δσ/E_{sec}), (0.005-0.006), (0.006-0.007), (0.007-0.008) and (0.008-0.009). The curves were analyzed to determine (a) the reduction in secant modulus with increasing compressive strain, and (b) the accumulation of residual strain with increasing number of cycles.

DATA ANALYSIS
The decrease in secant modulus with increasing strain for bovine trabecular bone was similar under monotonic and cyclic compression tests. The secant modulus decrease was relatively independent of the loading characteristics. This similarity in the secant modulus decrease under the two loadings suggests a strain-based failure criterion for trabecular bone. Residual strain accumulation with increasing number of cycles was divided into primary and secondary phases. The primary phase involved the initial cycles (~1-5) of cyclic loading. The primary residual strain was similar for cyclic compression tests at different normalized stresses. For a given specimen, the secondary residual strain accumulation rate (i.e. accumulation per cycle) was approximately constant and was less than that in the primary phase.

NUMERICAL MODEL
For any given cycle in the cyclic compression test, the maximum strain (ε_{max}) measured during that cycle is the sum of the residual strain at the beginning of the cycle (ε_{res}) and the strain increment during the cycle (Δσ/E_{sec}, where Δσ is the maximum stress in the cycle, and E_{sec} is the secant modulus). The residual strain can be divided into a primary (ε_{res}^{pr}) and a secondary (ε_{res}^{sec}) component,

\[ ε_{max} = ε_{res}^{pr} + N \frac{dε_{res}}{dN} + \frac{Δσ}{E_{sec}} \]  

(1)
where $N^{\text{sec}}$ is the number of cycles in the secondary phase and $d \varepsilon_{\text{res}}^{\text{sec}}/dN$ is the secondary residual strain accumulation rate. $d \varepsilon_{\text{res}}^{\text{sec}}/dN$ was a function of the maximum normalized stress, while $E_{\text{sec}}/E_0$ was a function of the maximum strain ($\varepsilon_{\text{max}}$). Incorporating $\varepsilon_{\text{max}} = f_1^{-1}(E_{\text{sec}}/E_0)$, $d \varepsilon_{\text{res}}^{\text{sec}}/dN = f_2(\Delta \sigma/E_0)$ in Eq. (1),

$$f_1^{-1}(E_{\text{sec}}/E_0) = \left\{ \varepsilon_{\text{res}}^{\text{pr}} + (N - N^{\text{pr}}) f_2(\Delta \sigma/E_0) \right\} + \left( \frac{\Delta \sigma/E_0}{E_{\text{sec}}/E_0} \right)$$

where, $N = N^{\text{pr}} + N^{\text{sec}}$, and $N^{\text{pr}}$ is the number of cycles in the primary phase.

The functions $f_1$ and $f_2$ varied from specimen to specimen. Upper and lower bounds for the normalized secant modulus ($E_{\text{sec}}/E_0$) vs. $N$ variation could be estimated by evaluating Eq. (2) for suitable bounds of functions $f_1$ and $f_2$. Bounding functions for $f_1$ were estimated using upper and lower bounds of the normalized secant modulus reduction with increasing strain during monotonic loading. For each of the four stress ranges, the upper and lower bounds of the $f_2$ function were estimated from experimentally determined residual strain accumulation rates. Given the bounds of $f_1$ and $f_2$, the bounding curves for the normalized secant modulus variation with increasing number of cycles were determined using Eq. (2).

RESULTS

The model predicts the bovine trabecular bone S-N curve, i.e. the number of cycles corresponding to a given loss of secant modulus at different normalized stresses. From Eq. (2):

$$N = \frac{f_1^{-1}(E_{\text{sec}}/E_0) - \varepsilon_{\text{res}}^{\text{pr}}}{f_2(\Delta \sigma/E_0)} + N^{\text{pr}}$$

$f_1$, $\varepsilon_{\text{res}}^{\text{pr}}$ and $N^{\text{pr}}$ were relatively insensitive to changes in normalized stress. The variation of secondary residual strain accumulation rate with normalized stress ($f_2$) was estimated from experimental data, and the number of cycles ($N$) at each normalized stress, corresponding to a 10% loss in secant modulus, was determined using Eq. (3). The upper and lower bounds of the S-N curves were determined using the bounds of $f_1$ and $f_2$. The bounds compared well with experimentally observed S-N data (Figure 1). The predicted endurance limit for bovine trabecular bone at $10^6$ cycles fell between $\Delta \bar{\sigma}/E_0$ values of 0.0032 and 0.0024.

**DISCUSSION**

The experimentally determined $E_{\text{sec}}/E_0$ vs. $N$ variation for different trabecular bone specimens were observed to lie within the bounds defined by the model. Considering that the inputs to the numerical model were (a) bounds of the $f_1$ function (determined from monotonic loadings), and (b) bounds of the $f_2$ function for each $\Delta \sigma/E_0$ range, the model suggests the possibility of effective estimation of $E_{\text{sec}}/E_0$ vs. $N$ plots based on a small number of cyclic tests, complimented by more economical monotonic compression tests, and reduces the need for long-term fatigue tests.

**REFERENCES**


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