Dynamic Loading and Biological Growth

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INTRODUCTION

Biological materials are complex structures that can undergo processes of growth (or resorption) in response to mechanical stimuli. Roughly speaking, these processes are of two kinds: (i) bulk growth, whereby the material particles retain their identity and experience a possible rearrangement (remodeling) with or without increase of mass; and (ii) surface growth, which may affect the very topology of the body (for example, by closing voids). Bulk growth, which is the subject of this paper, can be accompanied by mass diffusion (mass flux), but this effect is disregarded in the present work. One of the first complete models of biological growth was proposed by Hegedus and Cowin (1976). Although not explicitly stated, one of the limitations of this pioneering model is that the new mass finds a way to accommodate itself within the available space without altering the underlying tissue. In some models (e.g., Mullender et al. (1994)), the process of remodeling is accompanied by a change of material properties. A different point of view was advocated by Rodriguez et al. (1994), who introduced a model akin to that of nonlinear plasticity theory based on a multiplicative decomposition of the deformation gradient. A full thermodynamic treatment along those lines was proposed by Epstein and Maugin (2000). From this point of view, one can distinguish between "compliant" processes that can evolve naturally without the need for the intervention of active controlling mechanisms (entropy sinks) and those that cannot. In particular, we investigate the influence of a periodical loading and conclude that, within very general parameters, growth is stimulated in a compliant manner. The model is then incorporated into a finite element package and some examples are presented.

MODEL FORMULATION

As time (t) goes on, a stress-free archetype, representing the basic material constitution, is linearly mapped into each body point ($\mathbf{X}$) by means of an *implant* $\mathbf{P}(\mathbf{X}, t)$. The deformation gradient $\mathbf{F}$ has its usual meaning of mapping the chosen arbitrary fixed reference configuration to the current deformed configuration. The Piola stress $\mathbf{T}$ is then considered a function of the linear mapping between the stress-free state and the current deformed configuration. This mapping is the composition of $\mathbf{FP}$. In addition to the constitutive equations of the archetype, an *evolution law* for the implant $\mathbf{P}$ is sought that would satisfy thermodynamic requirements. In the absence of mass diffusion and of external entropy sinks, the Clausius-Duhem inequality in its local Lagrangian form is expressed as:

$$\rho_R \dot{\psi}_\rho + \rho_R \dot{\theta} s_\rho - T_i^I v^i{}_{,I} + \frac{1}{\theta} Q^I \theta_{,I} \leq 0$$

where: ρ_R denotes the density of the material in the reference configuration; ψ_ρ is the Helmholtz free-energy per unit mass; θ denotes the absolute temperature; s_ρ is the entropy per unit mass; T_i^I denotes the (mixed) components of the Piola stress $\mathbf{T}$; v^i is the velocity field; and Q^I are the components of the heat-flux vector in the reference configuration. A comma preceding

an index indicates partial differentiation with respect to the corresponding coordinate in the underlying (Cartesian) system of coordinates and a dot indicates time derivatives. Following the standard procedure, the residual inequality for the implant evolution is obtained as:

$$\dot{P}^I_\alpha P^{-\alpha}_J m_I^J \leq 0$$

where **m** is the Mandel stress defined as:

$$m_I^J = F^i_I T_i^J$$

A possible law for evolution of the implant **P** can thus be assumed as:

$$\dot{\mathbf{P}} = -kJ_P \, \mathbf{P} \, \mathbf{P}^T \, \boldsymbol{m} \, \mathbf{P}^{-T}$$

where k is a positive material constant and J_P is the determinant of the implant **P**.

EXAMPLE AND FINITE ELEMENT ANALYSIS INCORPORATION

An example is presented to show the behaviour of the evolution law under sinusoidal load and to validate a user subroutine finite element code that was written to incorporate this material model in ABAQUS. A ball of material is assumed to be subjected to a sinusoidal applied load $a(t)=a_0\sin(\omega t)$. The deformation gradient **F** and the implant **P** are equal to $h(t)\mathbf{I}$ and $q(t)\mathbf{I}$, respectively. The constitutive law for Piola stress in the reference configuration is taken as $\mathbf{T} = \mu J_P^{-1} (\mathbf{FPP}^T - \mathbf{F}^{-T})$ from which the following relation: $h^2 a = \mu/q^3 (h q^2 - 1/q)$ can be written. The evolution law suggested in the previous section can be used to write the following equation: $\dot{q} = -kq^4h^3a$. The two equations were solved for μ and k =1 using the Mathematica Package. The average value of q obtained is shown to be decreasing with time (Figure 1). This indicates that the model predicts that the material grows under the effect of the sinusoidal load as compared to a static load of the same magnitude. The same problem was solved using ABAQUS by incorporating a user subroutine, UMAT, for the calculation of the Cauchy stress as a function of the deformation gradient and the implant. The nine components of the implant were calculated at the end of each time increment using an Euler time integration scheme and were stored as solution dependent variables to be used in the next increment. The finite element analysis and the Mathematica package produced the exact results.

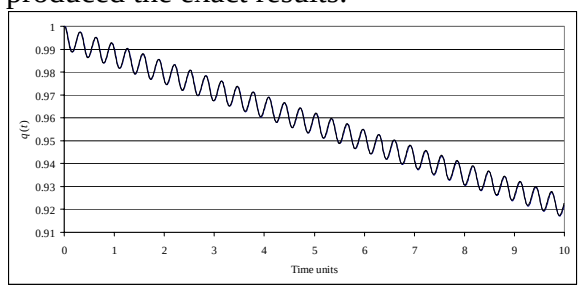

Figure 1. Evolution of the implant $q(t)$.

On the basis of this Benchmark test, more realistic examples will be presented.

SUMMARY

A model for the growth of biological material is formulated and shown to predict the growth of biological material under harmonic loading. The model was incorporated into a finite element package and can be easily calibrated against experimental data to mathematically model growth and resorption in biological tissues.

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