

# CRITERIA FOR DYNAMIC SIMILARITY IN BOUNCING GAITS

Sharon R. Bullimore<sup>1</sup>, Jeremy F. Burn<sup>2</sup> and J. Maxwell Donelan<sup>3</sup>

<sup>1</sup>Human Performance Lab, Faculty of Kinesiology, University of Calgary, Canada. <sup>2</sup>Departments of Anatomy and Mechanical Engineering, University of Bristol, UK. <sup>3</sup>Locomotion Lab, School of Kinesiology, Simon Fraser University, Canada. Email: sbullimore@kin.ucalgary.ca.

## INTRODUCTION

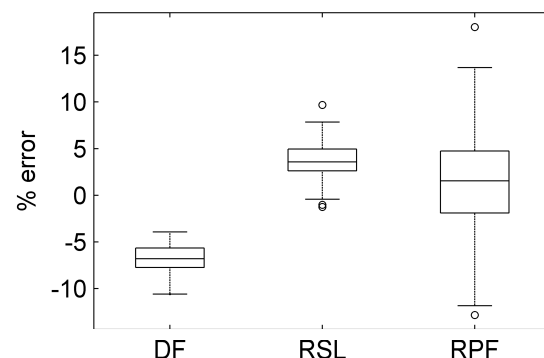
When animals of different sizes travel at speeds corresponding to equal values of the Froude number ( $u^2/gL$  where  $u$  is speed,  $g$  is the acceleration due to gravity and  $L$  is leg length) they tend to move in a dynamically similar manner – i.e. they use close to equal values of duty factor (DF), stride length relative to leg length (RSL) and peak vertical ground reaction force relative to body weight (RPF). Consequently, the Froude number has been widely used to define equivalent speeds for comparing animals of different sizes and to predict speeds in extinct species and on other planets (Vaughan and O'Malley, 2005). However, Donelan and Kram (2000) have demonstrated, using simulated reduced gravity, that equal Froude number does not guarantee dynamic similarity, even in perfectly geometrically similar individuals. Their results have cast doubt upon the validity of using the Froude number to define equivalent speeds or to make predictions.

It has been suggested that the planar spring-mass model may be useful for defining criteria for dynamic similarity (Blickhan, 1989; Blickhan and Full, 1993; Donelan and Kram, 2000). This simple model of bouncing gaits, such as run, trot and hop, consists of a mass bouncing on a linearly elastic spring. Here we use dimensional analysis of this model to obtain criteria for dynamic similarity in bouncing gaits and to determine why dynamic similarity does not

occur at equal Froude number in simulated reduced gravity.

## MODEL VALIDATION

We first needed to determine whether the model accurately predicts DF, RSL and RPF in normal and reduced gravity conditions. To do this, we compared its predictions to a subset of the data of Donelan and Kram (2000). The model provided excellent predictions of DF, RSL and RPF. All predictions were within 20% of measured values and more than 90% of predictions were within 10% of measured values (Fig. 1). Prediction error either decreased, or did not change, as gravity level decreased, indicating that this is also a good model of running in simulated reduced gravity.



**Figure 1:** Percent errors in model predictions of DF, RSL and RPF.

## DIMENSIONAL ANALYSIS

Buckingham's Pi-Theorem (Isaacson and Isaacson, 1975) indicated that the model parameters should be expressed as four dimensionless parameters (DPs). We

reduced this to three DPs by applying the constraint of a constant average speed of locomotion. The three DPs we used were: dimensionless leg stiffness ( $K$ ), Froude number and dimensionless vertical landing velocity ( $V_0$ ). All three of these DPs must be equal for bouncing gaits to be dynamically similar.

## SENSITIVITY ANALYSIS

In addition to identifying criteria for dynamic similarity, it is also essential to know their relative importance. To determine this, we used sensitivity analysis. 27,000 model simulations were conducted over a physiologically relevant parameter space. At each point in the parameter space, we determined the percentage changes in DF, RSL and RPF in response to a 5% change in each DP. The relative importance of the three DPs varied with speed and gait, but was typically of the same order of magnitude, indicating that all 3 DPs are important for dynamic similarity.

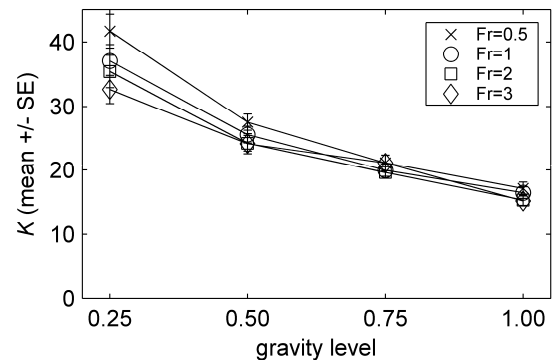
## REDUCED GRAVITY DATA

We reanalysed the simulated reduced gravity data of Donelan and Kram (2000) to calculate  $K$  and  $V_0$ . We found that the reason that humans do not run in a dynamically similar manner at equal Froude number in different gravity levels is that  $K$  decreases as gravity increases ( $p < 0.001$ ; Fig. 2).

## CONCLUSIONS

Equal Froude number is a necessary, but not sufficient, condition for dynamic similarity in bouncing gaits. Therefore, comparing animals at equal Froude number will not necessarily eliminate size effects. Such comparisons can still be useful, however, because equal Froude number makes dynamic similarity possible and so allows

fundamental differences in locomotion to be detected. Likewise, predictions of locomotion obtained by assuming dynamic similarity at equal Froude number will not always be accurate. Empirical evidence suggests that this approach can be effective in normal gravity however. More generally, dynamic similarity provides a powerful framework within which similarities and differences in locomotion can be interpreted.



**Figure 2:** Decrease in  $K$  with gravity level in humans running at equal Froude ( $Fr$ ) in different levels of simulated reduced gravity (multiples of Earth gravity).

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