

CONTINUUM-BASED MODEL OF SKELETAL MUSCLE

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INTRODUCTION

Knowledge of muscle forces during given activities can provide insight into muscle physiology, musculoskeletal mechanics, neurophysiology, and motor control. However, currently available methods for clinical examination or instrumented strength testing only provide information regarding muscle groups. Musculoskeletal models are typically needed to calculate individual muscle forces.

The objective of this study is to develop a continuum-based constitutive model to describe the three-dimensional mechanical response of muscle under isochoric (volume-preserving) passive and active deformation conditions within a thermodynamic framework. The model must account for transversely-isotropic material symmetry and explicitly state the strain-energy associated with the active and passive responses of the tissue.

CONSTITUTIVE MODELING

It has been reported that skeletal muscle tissue exhibits the same mechanical behavior on both the muscle fiber- and sarcomere-associated length scales (Zajac 1989). Therefore, the mechanical behavior of skeletal muscle tissue can be modeled as a continuous and homogenous effective continuum with mathematically-defined properties and symmetry on multiple length-scale levels (Fig. 1).

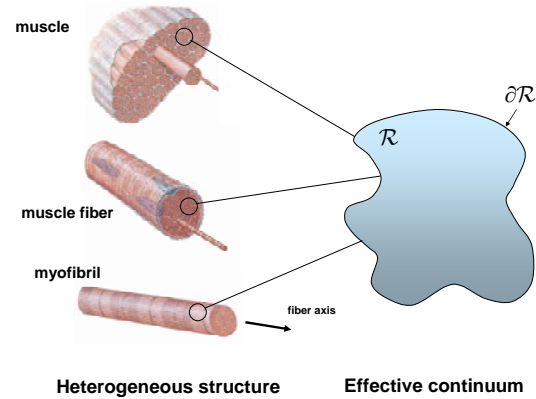


Figure 1: Effective Continuum assumption for all scale levels of skeletal muscle tissue (Netter medical illustration used with permission of Elsevier. All rights reserved.)

Following a similar approach developed for material response with internal variables (Coleman and Gurtin 1967), the strain-energy density due to the mechanical deformation, actin/myosin overlap, and muscle activation was formulated as

$$\Psi = \frac{\mu}{4} \left[\frac{1}{\alpha} (L_1^\alpha - 1) + \frac{1}{\beta} (L_2^\beta - 1) \right] + \frac{\gamma}{2} q \phi J_4 \quad (1)$$

where μ , α , β , and γ are material constants; L_1 , L_2 , and J_4 are scalar invariants of the Green deformation tensor \mathbf{C} and the structural tensors for transverse isotropy (Itskov and Aksel 2004); q is the scalar muscle activation parameter; and ϕ is the actin/myosin overlap parameter which is related to the active component of the strain-energy density. The bracketed and last term in Equation (1) represent the passive and active responses of the muscle tissue, respectively. The parameter $q = 0$ when the

muscle is unactivated and $q = 1$ when fully activated. The parameter $\phi = 1$ in the state of maximized cross-bridging. The constitutive response is given by

$$\mathbf{S} = \frac{\mu}{2} \left[(L_1^{\alpha-1}) \tilde{\mathbf{M}} - (L_2^{\beta-1}) \mathbf{C}^{-1} \tilde{\mathbf{M}} \mathbf{C}^{-1} \right] + \gamma q \phi \mathbf{M}_1 \quad (2)$$

where \mathbf{M}_1 and \mathbf{M} are structural tensors and \mathbf{S} is the 2nd Piola-Kirchhoff stress tensor.

RESULTS AND DISCUSSION

Characterization of this constitutive model using data from the literature (Jenkyn et al. 2002) results in the following values of material parameters: $\mu = 300$ kPa, $\alpha = 11.1$, $\beta = 5.3$, and $\gamma = 232$ kPa. A comparison of the total stress response for the proposed model and the Jenkyn model is shown in Figure 2 for the range of longitudinal strains (Green strain) of $-0.3 < E_{11} < 1.3$, where the x_1 -axis is aligned with the fiber axis. The inset of Figure 2 is the small-strain region of the plot. The nonlinearity of the data in the small-strain region resembles the expected combined response. An inflection point exists at $E_{11} \approx 0.1$ where the active tension is decreasing and the passive tension is increasing with increasing E_{11} .

The combined passive and active stresses for the longitudinal extension show excellent agreement with the Jenkyn model. The Jenkyn model appears to be the most comprehensive model for skeletal muscle tissue currently available in the literature in that it incorporates a large range of

deformation modes, albeit in two dimensions. Therefore, an accurate mathematical model of skeletal muscle tissue has been established that advances the state of muscle modeling. A set of three-dimensional experimental data is now needed to further validate the proposed model.

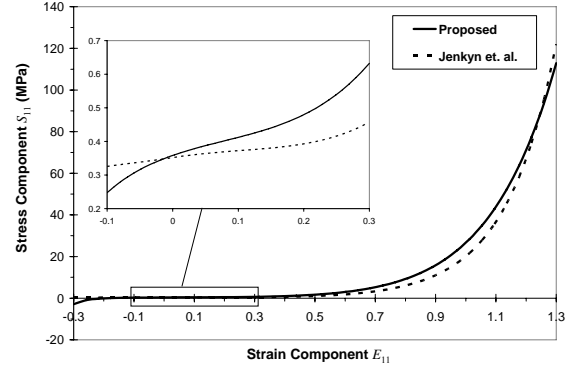


Figure 2: Total stress vs. strain diagram for fully activated skeletal muscle.

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ACKNOWLEDGEMENTS

This publication was made possible by Grant Number R01HD31476 from NIH.