A METHOD TO DETERMINE WHETHER A MUSCULOSKELETAL MODEL CAN RESIST ARBITRARY EXTERNAL LOADINGS WITHIN A PRESCRIBED RANGE

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INTRODUCTION

Musculoskeletal models often use mathematical representations of the anatomy to compute muscle lengths and moment arms [1]. It is possible to create a model that will not satisfy mechanical conditions of equilibrium for a set of external loading conditions. Such a model would be "unloadable." A model that has a solution for any externally applied force that lies in a prescribed range will be termed "loadable" here. The purpose of this project is to develop an efficient method for testing whether a musculoskeletal model geometry is loadable.

METHODS

Consider a static musculoskeletal model with joints and muscles, along with an external force **F** applied to the model. Static equilibrium for the model can be written in the standard matrix notation

$$\mathbf{Ax} = \mathbf{b} \qquad (1)$$
$$\mathbf{x} \geq \mathbf{0}$$

where **A** is a matrix whose columns each correspond to unit torque for each muscle, **x** is a vector containing force magnitudes for each muscle, and **b** is a vector containing the torque generated by the external force **F** and the torques from the masses of the appendages. Additional rows are added to **A** for the degrees of freedom provided by additional joints in the model. The system is considered arbitrarily loadable if there exists a solution **x** to (1) for any arbitrary **F**.

In order to determine the loadability of a model for a specific and continuous range of forces, the user can specify an arbitrary number of vectors $\mathbf{h}_k$ so that these vectors describe the boundary of a convex hull of externally applied force vectors. Every force vector **F** that lies in the convex hull can be written as a non-negative linear combination of the vectors $\mathbf{h}_k$:

$$\mathbf{F} = \sum_{k=1}^{q} \lambda_k \mathbf{h}_k \qquad (2)$$

$$\sum_{k=1}^{q} \lambda_k = 1 \quad \text{and} \quad \lambda_k \geq 0 \;\; (k = 1, 2, \ldots, q)$$

where λ_k is an arbitrary non-negative scalar multiplier, the $\mathbf{h}_k$ vectors specify the boundary of the user-chosen convex hull of external forces, and q is the number of $\mathbf{h}_k$ vectors.

The system is determined to be loadable for all vectors that lie in the convex hull if one or more solutions, $\mathbf{y}_k$, to the following equations

$$\mathbf{Ay}_k = \boldsymbol{\beta}_k \qquad (3)$$
$$\mathbf{y}_k \geq \mathbf{0} \qquad (k = 1, 2, \ldots, q)$$

exist, where $\boldsymbol{\beta}_k$ is a vector containing the torque generated by each $\mathbf{h}_k$ vector as the external force, along with torques from the masses of the appendages. In other words, each equation in (3) is the same as in (1), except that $\mathbf{h}_k$ is used for the externally applied force vector. Note that (3) actually consists of q equations, each using a different $\boldsymbol{\beta}_k$. By solving (3) for each $\boldsymbol{\beta}_k$, loadability is determined for all the possible force vectors that lie in the convex hull specified by the $\mathbf{h}_k$ vectors. If one or more solutions to (3) do not exist, the system is unloadable since the equations in (3) are merely using each $\mathbf{h}_k$ as the external force vector.

However, for the purposes of determining loadability of a musculoskeletal model, only the feasibility of a solution $\mathbf{y}_k$ to (3) needs to be ascertained. If there are p degrees of freedom in the model, feasibility can be determined by solving the linear program:

$$\text{minimize} \quad \sum_{j=1}^{p} \mathbf{u}_j + \sum_{j=1}^{p} \mathbf{v}_j \qquad (4)$$
$$\text{subject to} \quad \mathbf{Ay}_k + \mathbf{u} - \mathbf{v} = \boldsymbol{\beta}_k$$
$$\mathbf{y}_k \geq \mathbf{0}, \mathbf{u} \geq \mathbf{0}, \mathbf{v} \geq \mathbf{0} \qquad (k = 1, 2, \ldots, q)$$

The formulation in (4) can be solved using the revised simplex method [2], a matrix-based implementation of the generalized simplex method [3]. Each formulation in (3) has a solution if and only if each one in (4) has a solution with $\mathbf{u} = \mathbf{v} = \mathbf{0}$. Consequently, solutions to (4) with $\mathbf{u} = \mathbf{v} = \mathbf{0}$ are necessary and sufficient for loadability of the model.

RESULTS AND DISCUSSION

The method of determining loadability was applied to two examples involving a three-dimensional shoulder model: one example that was determined to be loadable, and one that was determined to be unloadable. The shoulder model (Figure 1) was constructed using Repository 6.2 of the AnyBody Modeling System™ version 3.0, and the loadability method and resulting analysis was implemented in MATLAB.

The AnyBody Modeling System uses inverse dynamic analysis to determine the muscle configuration of its models [4]. For a given posture and externally applied force, the software produces a matrix similar to **A** in (1), and a vector similar to **b** in (1). The appropriate entries of this matrix and vector were used to solve for muscle force activity using (4). The static posture used in the analysis is shown in Figure 1, and consists of a glenohumeral abduction angle of $\pi/6$, an elbow flexion angle of $\pi/2$, and an elbow pronation angle of $5\pi/18$. The convex hull of externally applied forces is specified by three vectors applied medially to the elbow, in addition to the zero vector applied at the elbow, as shown in Figure 1. The AnyBody global coordinate convention is such that $+x$ is forward from the body, $+y$ is up, and $+z$ is lateral to the right of the body. The three non-zero vectors describing the convex hull are at angles of $\pi/6$ to the z-axis.

Since the external forces are producing adduction of the shoulder, removing all the deltoid and supraspinatus muscles may cause the model to be unloadable since those muscles are the major abductors. The method described by (4) was applied, and it was determined that the shoulder model without the deltoid and supraspinatus muscles was in fact loadable. Although only the existence of solutions to (3) is necessary to determine loadability for the convex hull, a solution was solved for each of the three non-zero force vector vertices of the convex hull. The solution for each of these vertices used an infraspinatus segment, a triceps (medial head) segment, and a pronator teres caput humeral segment to provide the necessary abductive force. Each solution used a different amount of force for each of those muscles, but all three solutions activated those same muscles. Since the model specified by (1) does not impose an upper limit on the amount of force each muscle can produce, our mathematical method found a solution

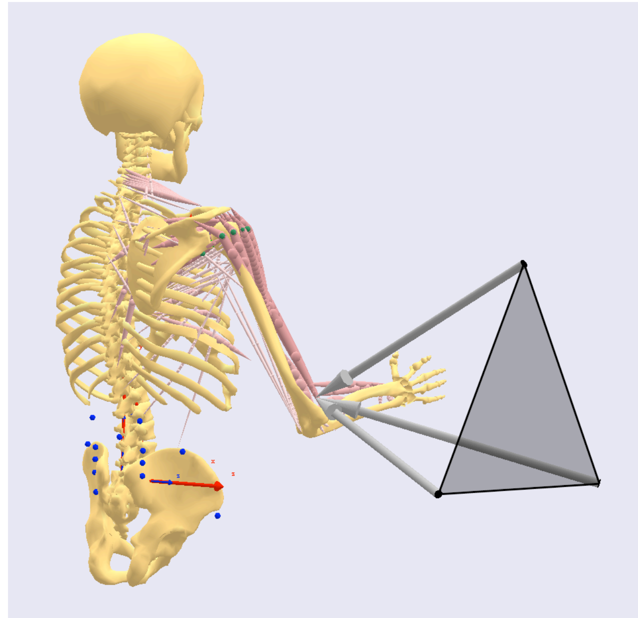

Figure 1. Anybody shoulder model used in the examples.

to resist the convex hull of external forces by utilizing weaker shoulder abductors such as the infraspinatus combined with the triceps. The pronator teres muscle was activated in order to maintain the elbow flexion angle of $\pi/2$, and elbow pronation angle of $5\pi/18$ in the model posture. Hence, this model was determined to be loadable for the specified convex hull of forces.

Next, in addition to taking out all the deltoid and supraspinatus muscles, all the infraspinatus muscles and the medial heads of the triceps muscles were removed. After applying the loadability algorithm, solutions to (4) with $\mathbf{u} = \mathbf{v} = \mathbf{0}$ could not be found for each vector defining the vertices of the convex hull. Since (4) could not be solved with $\mathbf{u} = \mathbf{v} = \mathbf{0}$, the shoulder model was determined to be unloadable. Removing these additional muscles eliminates all muscles with an abduction moment, and the shoulder is not able to abduct against the external forces.

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