A CONTINUOUS METHOD TO QUANTIFY STRESS-STRAIN BEHAVIOR OF BIOLOGIC MATERIALS

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INTRODUCTION

Mathematical models of biologic structures provide opportunities to understand complex interactions and simulate treatment outcomes. Accurate estimates of the material properties of biologic soft tissues are critical to the fidelity of these models. Biologic soft tissues have a nonlinear mechanical behavior characterized by an exponential toe region followed by a linear elastic region [1-4]. Traditionally, two curves are independently fit to a set of data [5]. Thus, the two portions of the modeled stress-strain curves may not exhibit both C₀ and C₁ continuity, and could inaccurately estimate the true material properties.

Here, a Continuous Method (CM) is introduced that enforces continuity in both the stress-strain curve and its derivative by optimizing the fit for both regions simultaneously. Its performance is evaluated and compared to the traditional Piecewise Method (PM).

METHODS

Both the traditional PM and the CM were implemented via custom MATLAB® (The MathWorks, Natick, MA) code. Both methods were used to analyze two data sets: ideal data and measured stress-strain data.

Piecewise Method (PM) -- The PM employs a linear curve fit to the high-strain end of the stress-strain data. Data is included in the linear region until the R² of the fitted line dips below an a priori determined threshold value (R²_cutoff= 0.99 in this work) [5]. The remaining lower strain points are fitted with an exponential curve [3].

Continuous Method (CM) -- A mathematical model that contains an exponential region and a linear region maintains a continuous elastic modulus E by defining the slope of the linear portion as the slope of the stress-strain curve at the transition point (p, q) between the exponential and linear regions. That is,

\[ \sigma = \begin{cases} 
A(e^{B\varepsilon} - 1) & \forall \varepsilon \leq p \\
E(p,q)(\varepsilon - p)+q & \forall \varepsilon > p 
\end{cases} \]

where \( E(p, q) \) is the elastic modulus at point \( (p, q) \).

The optimal parameters values for \( A, B, \) and \( p \) are computed by minimizing the mean square error \( (MSE) \). This yields the least-squares error between the modeled function and the experimental data. The optimization simultaneously fits the exponential section and linear section, and the location of the transition point, \( p \), is included in the optimization.

Data Sets -- The ideal data set used an exponential curve that smoothly transitioned to a line,

\[ \sigma = \begin{cases} 
0.200(e^{0.05\varepsilon} - 1) & \forall \varepsilon \leq 0.10 \\
232(\varepsilon - 0.10)+6.42 & \forall \varepsilon > 0.10 
\end{cases} \]

The curve had C₀ and C₁ continuity and a known transition point at \((0.10, 6.42)\). The measured data was collected from a sample of porcine lateral meniscus undergoing tensile testing.

RESULTS AND DISCUSSION

Ideal Data -- The PM generated a stress-strain curve with a discontinuity of 3.03 MPa at the transition point, or 319% of the actual stress magnitude at this location (Figure 1). The discontinuity in its derivative resulted in a modulus increase from 25.7 MPa to 206 MPa at the transition. The estimates for parameters \( A \) and \( B \) had high errors (Table 1). However, the error in estimating the elastic modulus was lower, 11.2%.
The PM was also inaccurate at estimating the location of the actual transition point, underestimating the both stress and strain (Table 1).

In contrast, the CM generated an unbroken curve through the transition point (Figure 1) and without discontinuity in modulus. The parameters A and B were estimated within 7.00% and 2.57% of the known values, and the elastic modulus, E, within 0.431%. The location of the transition point was within 2% of the known strain and stress.

*Measured Data* -- Application of the PM resulted in a discontinuity of 1.9 MPa at the transition. The model closely approximated the measured data in the exponential region, but it deviated from the actual measured data points in the linear region (Figure 2). The CM yielded a curve that more closely tracks the measured data points and had a MSE over 20 times lower that obtained using the Piecewise Method (Table 2).

**CONCLUSIONS**

The CM dramatically reduced errors in estimating the parameters A, B, E, p, and q when applied to the ideal data with known solutions. When applied to measured data without a known solution, the CM had a much better fit indicated by the reduced MSE. The PM yielded discontinuities in both the stress strain curves and the modulus. In contrast, the CM resulted in a continuous curve and modulus.

The Continuous Method may be a more accurate way to estimate material parameters of soft biological tissues. A key element of the Continuous Method is to simultaneously solve a problem that had historically been solved sequentially. Using this simultaneous approach may broadly impact the fields of engineering modeling and biomechanics.

**REFERENCES**


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